## scịence-3D

## DESERT BATTLE: NINJA RAT VS RATTLESNAKE

In this packet, sample student answers are provided in red and notes to teachers are in blue.
In this STEM Project, students will explore the relationship between surface area and volume and how it is impacted by the shape of objects. They will reinforce math skills and apply their discoveries to understand how natural and engineered structures function. Then, they will apply their understanding to design solutions to several engineering challenges.

## ACTIVITY I: MATH TIME!

Note: If students are familiar with surface area and volume calculations, you can skip this activity. However, you might want to lead a group discussion using the graph of how surface area and volume scale with size of different shapes before moving to the next activity. You could also have different students or groups fill in the table for one of the shapes and then work with other students and groups to complete the table and make comparisons.


Figure 1. Three dimensional shapes

To limit time spent on this activity, there are only a few calculations that students are asked to make - for cubes and cuboids. Then, students are asked to investigate relationships within graphs. Consider having students generate graphs for different shapes on their own. To limit the number of calculations they need to complete, have them use the surface area and volume calculators at:
https://www.calculator.net/surface-area-calculator.html and https://www.calculator.net/volume-calculator.html. Divide students into groups and assign groups different shapes. Have them generate curves of how surface area and volume and the ratio of surface area to volume change with the size of the shape. Then, have the students compare and contrast their results with other groups.

Before we start working on surface area and volume, let's do some calculations. How does the size and shape of an object influence its surface area and volume? There are many different shapes that we could investigate shown in Figure 1.

Start with a cube.

a

To calculate the surface area, you need to add up the area of all the surfaces. Since there are six surfaces and each surface is the same size, we can calculate surface area by:

Surface area (cube) $=6 \cdot a^{2}$ where " $a$ " is the length of $a$ side.
Volume is calculated by multiplying the length, width, and height of the cube. Since each of those measurements are the same in a cube, volume is:

Volume (cube) $=a \cdot a \cdot a$ or $a^{3}$

1. Use the information above to complete Table 1. To calculate the Surface Area: Volume ratio, divide Surface Area by Volume.

Table 1. Surface area, volume, and surface area : volume ratio for cubes of five different sizes

| Side Length <br> $(\mathrm{cm})$ | Surface Area <br> $\left(\mathbf{c m}^{\mathbf{2})}\right.$ | Volume <br> $\left(\mathbf{c m}^{\mathbf{3}}\right)$ | Surface Area: <br> Volume Ratio |
| :---: | :---: | :---: | :---: |
| 1 | 6 | 1 | 6 |
| 2 | 24 | 8 | 3 |
| 5 | 150 | 125 | 1.2 |
| 10 | 600 | 1,000 | 0.6 |
| 20 | 2,400 | 8,000 | 0.3 |

2. As the length of a side increases, does surface area or volume increase more quickly? Cite evidence from Table 1 to support your answer.

Volume increases much faster as the length of the side increases. Increasing the size of a side from 1 cm to 20 cm results in the surface area going up 400 times (or $2,394 \mathrm{~cm}^{2}$ ) while volume
goes up 8,000 times $\left(7,999 \mathrm{~cm}^{3}\right)$.
3. Describe what the ratio of surface area to volume shows. Do larger values mean there is more surface area for a particular volume or less surface area relative to the object's volume?

The surface area: volume ratio tells you how much surface area there is for a particular volume. The larger the value of the ratio, the more surface area there is relative to volume. Lower values mean there is more volume. This question is primarily here to test whether students are able to use the concept of ratios. Rather than giving credit for being correct, this might be a good place to have a class discussion to test and reinforce learning. For example, ask the class if a very round object would have a large ratio relative to a very large and flat object (the round object would have a lower ratio).

How about a rectangular prism? It's similar to the cube but not all the sides are the same length. We need to add the area of each of its six surfaces together. So, we need to measure the length

(I), width (w), and height (h) of the rectangular prism.

Surface Area (Rectangular prism) $=(2 / w)+(2 / h)+(2 h w)$
and
Volume (Rectangular prism) $=1 w h$
4. Use the information from the previous page to complete Table 2. Round answers to the nearest tenth.

Table 2. Surface area, volume, and surface area : volume ratio for rectangular prisms of five different sizes

| Length, width, <br> height $(\mathbf{c m})$ | Surface Area <br> $\left.\mathbf{( c m}^{2}\right)$ | Volume <br> $\left(\mathbf{c m}^{3}\right)$ | Surface Area: <br> Volume Ratio |
| :---: | :---: | :---: | :---: |
| $1,1,2$ | 10 | 2 | 3.3 |
| $1,2,3$ | 22 | 6 | 3.7 |
| $4,5,6$ | 148 | 120 | 1.2 |
| $8,10,12$ | 592 | 960 | 0.6 |
| $10,20,33$ | 2,380 | 6,600 | 0.4 |
| $15,15,32$ | 2,370 | 7,200 | 0.3 |

5. Describe the pattern of surface area and volume in the rectangular prism as its size and shape change. Hint: Compare the last two rows that have similar surface area but different shapes. Use the data in Table 2 to support your answer.

Larger rectangular prisms have greater surface areas and volumes, but volume increases more quickly in most cases. The dimensions of the shapes do matter. For instance, changing the dimensions of shape from $10 \mathrm{~cm} \times 20 \mathrm{~cm} \times 33 \mathrm{~cm}$ to $15 \mathrm{~cm} \times 15 \mathrm{~cm} \times 23 \mathrm{~cm}$ decreases surface area but increases volume.


Figure 2. Surface area : volume ratio of five different shapes
(All shapes had the same surface area. The rectangular prism measured $10 \mathrm{~cm} \times 20.5 \mathrm{~cm} \times 1 \mathrm{~cm}$.)
6. Based on Figure 2, describe what shapes have the highest volume for a certain surface area. Which shapes have the greatest surface area?

Spheres and capsules have the greatest volume relative to their surface area. The rectangular prism has a much greater surface area.

## ACTIVITY 己: HOW DOES SHAPE INFLUENCE FUNCTION?

Before we start putting surface area and volume concepts to work, let's think about what processes and factors are most influenced by surface area and which are most influenced by volume!

1. In Table 3 choose whether the listed features are influenced by surface area or volume.

Table 3. Features influenced by surface area or volume

| Feature | Influenced by: |
| :--- | :--- |
| Amount of light collected by a leaf in the desert | Surface Area |
| Amount of water lost by a leaf in the desert sun | Surface Area |
| Amount of water stored by a leaf | Volume |
| Amount of sun captured by an object | Surface Area |
| Amount of heat stored by an object | Volume |
| Amount of waste or nutrients moving across a cell membrane | Surface Area |
| Amount of waste produced by a cell or body | Volume |
| Amount of packaging material needed on the outside of an object | Surface Area |
| Amount of material inside a package | Volume |
| The rate at which the cover of a pill of medicine will dissolve | Surface Area |
| The amount of medicine in a pill | Solume |
| Speed that water will weather a rock | Surface Area Area |

Use the information in Table 3 to answer the following questions.
2. Imagine you want to cook a tray of brownies as fast as possible and have to use all of the batter you made. Would you choose a baking pan that is $20 \mathrm{~cm} \times 20 \mathrm{~cm} \times 5 \mathrm{~cm}$ or $30 \mathrm{~cm} \times 40 \mathrm{~cm} \times$ 5 cm ? Why did you choose this tray? Be sure to include the terms surface area and volume in your answer.

I would choose the $30 \mathrm{~cm} \times 40 \mathrm{~cm} \times 5 \mathrm{~cm}$ pan because it has more volume and the brownies would spread out more. They would also have more surface area than if I used the $20 \mathrm{~cm} \times 20$ $\mathrm{cm} \times 5 \mathrm{~cm}$ pan. Greater surface area, and less thick batter, would result in the brownies cooking faster.
3. Based on your answers in Table 3, predict why the size of cells is similar in large organisms and small organisms. Why don't we find huge cells in nature? Hint: Think about the processes that cells must carry out and how they are influenced by surface area and volume. Refer to Tables 1 and 2.

Cells get energy and nutrients based on how much surface area they have. The amount of waste they can release is based on their surface area. The amount of energy they need and the waste they produce is related to their volume. So, as cells get too big their volume increases too much compared to their surface area. They wouldn't get enough food or remove enough waste.
4. In the desert, plants have shapes and features that are adapted to retaining water. Desert plants do not have big thin leaves like plants in cooler climates. Instead, the leaves are thicker. They have less surface area and greater volume. Describe why you think this occurs.

In the desert, plants can't afford to lose water. Big thin leaves have a lot of surface area and not much volume. They couldn't store as much water and would lose water more quickly than leaves with more volume and less surface area.

## ACTIVITY 3: CREATING SOLUTIONS

Use what you have learned to design and draw solutions to the following questions. Label each drawing with key features and describe how the surface area and volume of your design is important to its function.

1. Create a solar panel that collects maximum sunlight using the minimum amount of building materials.

Drawing of solar panels should be thin with large surface areas. Descriptions should include that surface area will maximize the sunlight intercepted, but the small volume will mean that fewer building materials will be needed.
2. Design a building that could be used in the desert to prevent overheating during the day and quickly cool down at night. You have materials to build walls that measure 25 units on each side. What would be the building dimensions? Remember, you need four walls, a roof and a floor! Draw your design. Then describe why you drew this shape.

Students might have a variety of designs. They may draw a high-volume house to keep from getting too much heat. They might suggest a smaller surface area of windows, since they would heat up quickly. They might think of ways to let more air flow through at night or change the surface area to cool down.

Extend the lesson: Have students design a physical model of their building using construction paper or other materials.

Extend the lesson further: Investigate surface area and volume using experiments. You will need aluminum foil, a freezer, graduated cylinders and beakers of water.

Step 1. Have students create molds of different sizes and shapes that they think will have different surface area and volumes. To keep it simple, have students make shapes that have flat bottoms and sides.
Step 2. Have students label their molds and create a data table in which they can record information about their molds.
Step 3. Have students measure the dimensions of their molds.
Step 4. Have students record the volume of water in a graduated cylinder. Have them pour water into a mold.
Step 5. Each mold should have the same amount of water. Repeat for all molds.
Step 6. Have students calculate the surface area of the molds.
Step 7. Freeze the molds.
Step 8. Have students make predictions about which ice shapes will melt the quickest.
Step 9. Have students fill beakers up with water that is the same volume and temperature. Have students prepare a timer.
Step 10. Remove the ice from the freezer, and have students measure the dimensions of the ice. Then have them put the ice from each mold into its own beaker.
Step 11. Have students time how long each ice shape takes to melt.
Step 12. Have students compare their results to their predictions and discuss their results.

